

# Interacting Modified Holographic Dark Energy in Kaluza-Klein Universe

M. Sharif <sup>\*</sup> and Abdul Jawad <sup>†</sup>

Department of Mathematics, University of the Punjab,  
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

## Abstract

The interaction of modified holographic dark energy and dark matter with varying  $G$  in flat Kaluza Klein universe is considered. Further, we take infrared cutoff scale  $L$  as future event horizon. In this scenario, equations of state parameter as well as evolution are explored. We also check the validity of the generalized second law of thermodynamics. It is interesting to mention here that our results show consistency with the present observations.

**Keywords:** Kaluza-Klein cosmology; Modified holographic dark energy; Dark matter; Generalized second law of thermodynamics.

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## 1 Introduction

It is believed that our universe has entered in an accelerated expansion phase and experienced a large negative pressure (Riess et al. 1998; Perlmutter et al. 1999; Fedeli et al. 2009; Caldwell and Doran 2004). The force responsible for driving the universe apart is some cosmological antigravity substance

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<sup>\*</sup>msharif.math@pu.edu.pk

<sup>†</sup>jawadab181@yahoo.com

known as the mysterious dark energy (DE). Although 75 percent of the mass-energy content of our universe contains this type of DE but its nature is still speculative. One usually characterizes the DE phenomena with equation of state (EoS) parameter  $\omega$  (the ratio of pressure to energy density):  $\omega$  lying in the range  $(-1, -1/3)$  describes quintessence DE era,  $\omega = -1$  represents DE due to cosmological constant consistent with current observations (Davis et al. 2007), while it may be  $\omega < -1$ , the phantom case (which can lead to a future unavoidable singularity of spacetime).

The simplest candidate of DE is the cosmological constant but it faces some serious problems (Weinberg 1989, 2000) like 'fine tuning problem' (unusual small value) and 'coincidence problem' (why DE and DM are of the same order today even though universe is expanding?). The dynamical DE scenario is an alternative way to alleviate or even solve these problems. So far, a plethora of dynamical DE models has been proposed by physicist which are classified into two categories: scalar field models containing quintessence (Ratra and Peebles 1988), phantom (Caldwell 2002; Carroll et al. 2003), k-essence (Armendariz-Picon et al. 1999, 2001; Chiba, et al. 2000), tachyon (Padmanabhan 2002; Bagla et al. 2003), quintom (Feng, et al. 2005) and the interacting DE models (interaction of DE with DM) including family of Chaplygin gas (Kamenshchik et al. 2001; Bento et al. 2002; Zhang et al. 2006), braneworld (Deffayet et al. 2002; Sahni and Shtanov 2003), holographic DE (HDE) (Hsu 2004; Li 2004), new agegraphic DE models (Cai 2007) etc. These possibilities reflect the indisputable fact that the true nature and origin of DE has not been convincingly explained yet.

In the aforementioned candidates of DE, holographic dark energy is one of the marvelous attempts to examine the nature of DE in the framework of quantum gravity. It is based on holographic principle (Susskind 1995) which states that *all the information relevant to a physical system inside a spatial region can be observed on its boundary instead of its volume*. The main feature of this DE is that it links DE density to the cosmic horizon (a global property of the universe). Cohen et al. (1999) argued that the quantum zero-point energy of a system with size  $L$  (or infrared (IR) cutoff) should not exceed the mass of a black hole with the same size, i.e.,  $L^3 \rho_v \leq LM_{pl}^2$  (where  $\rho_v$  indicates the quantum zero-point energy density and  $M_{pl} = (8\pi G)^{-1/2}$  is the reduced Planck mass). Its original form is defined as (Hsu 2004)

$$\rho_\Lambda = \frac{3m^2}{8\pi GL^2},$$

where  $m$  is constant and  $G$  is the gravitational constant which is taken as a function of time.

The variation of  $G$  with time is the natural consequence of Dirac' "Large Number Hypothesis" (LNH) (Dirac 1938). Following this hypothesis, many efforts have been made through different aspects to obtain physical results about the universe. The idea of Brans-Dicke (BD) theory (Brans and Dicke 1961) and its generalization to other forms of scalar-tensor theories (Bergmann 1968; Wagoner 1970; Nordtvedt 1970) arose from variable- $G$  theories in which gravitational constant is replaced by a scalar field coupling to gravity through a new parameter. Further, it was shown through different observations in favor of hypothesis that  $G$  could be varied as a function of time or equivalently of the scale factor (Umezue et al. 2005; Nesseris and Perivolaropoulos 2006; Biesiada and Malec 2004). Recent work (Setare 2006a, 2006b; Jamil et al. 2009) shows keen interest to take interacting HDE with or without varying  $G$  in order to explain the current status of the universe. Some people (Setare 2006c; Setare and Shafei 2006; Setare and Vagenas 2008) have also explored the generalized second law of thermodynamics (GSLT) with interacting HDE. Also, this work was extended in different modified gravities like  $f(R)$  theory (Setare 2008), BD theory (Setare and Jamil 2010a), Gauss-Bonnet theory (Setare and Jamil 2010b), Horava-Lifshitz theory (Setare and Jamil 2010c) and Kaluza-Klein theory (Sharif and Khanum 2011).

Kaluza-Klein (KK) theory is an extra dimensional theory which grew out of an attempt to couple gravity and electromagnetism (Kaluza 1921, Klein 1926). Further development classified this theory into two versions such as compact (fifth dimension is length like and it should be very small) and non-compact forms (fifth dimension is mass like) (Wesson 1984, 1999; Bellini 2003). The non-compact KK theory is the consequence of well-known Campbell's theorem in which one cannot introduce any matter into five dimensional ( $5D$ ) manifold by hand because matter appears in four dimensions induced by the  $5D$  vacuum theory (Wesson 1984, 1999; Bellini 2003). However, original KK idea became the base of other extra dimensional theories in different aspects like string theory (Polchinski 1998), brane models (Clifford 2003) and super gravity (Wess and Bagger 1992). The version of HDE in extra dimension is known as modified holographic dark energy (MHDE) (Gong and Li 2010) which can be derived from mass of black hole in  $N + 1$  dimensional spacetime (Myers 1987). Some authors (Gong and Li 2010, Liu et al. 2010) found interesting results about the evolution of the universe by considering MHDE model. Recently, Sharif and Khanum (2011) have dis-

cussed the interacting MHDE with DM in KK cosmology by taking IR cutoff as Hubble horizon and also explored that GSLT holds generally without any assumption in this scenario.

The selection of IR cutoff  $L$  is a crucial problem inherited in the original version of HDE model which could be taken as either Hubble horizon  $H^{-1}$ , or particle horizon or future event horizon. Hsu (2004) and Li (2004) showed that if  $L$  is taken as either Hubble horizon  $H^{-1}$ , or particle horizon then the DE model is not compatible with the current observational data. However, it was suggested by Li (2004) that future event horizon is the best choice for  $L$ .

This work is an extension of Sharif and Khanum (2011) with IR cutoff as future event horizon and also varying  $G$ . Equations of state parameter as well as evolution are formulated. We also investigate the validity of GSLT in this scenario. The paper is organized as follows: Section 2 is devoted to the formulation of interacting MHDE with DM and its discussion. In section 3, we study the validity of GSLT for interacting models. In the last section, we summarize our results.

## 2 Interacting Modified Holographic Dark Energy with Varying $G$

In this section, we discuss the interaction of MHDE with dark matter in Kaluza-Klein cosmology by taking  $G$  as a function of cosmic time  $t$ . We evaluate EoS parameter and evolution of MHDE. The non-flat KK universe (Ozel et al. 2010) is given by

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2)d\psi^2 \right], \quad (1)$$

where  $a(t)$  is the dimensionless scale factor measuring the expansion of the universe,  $k = -1, 0, 1$  is the curvature parameter for the open, flat and closed universe respectively. Equations of motion corresponding to (1) for flat KK universe are

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G(t)}{6} \rho, \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G(t)}{6} (\rho + 2p). \quad (3)$$

Here dot represents differentiation with respect to cosmic time  $t$  and  $p = p_\Lambda$ ,  $\rho = \rho_\Lambda + \rho_m$  are the pressure and density respectively (where subscripts  $\Lambda$  and  $m$  denote DE and DM respectively). Also, we define  $H \equiv \dot{a}/a$  as the Hubble parameter which estimates the expansion rate of the universe.

In this scenario, the fractional energy densities are

$$\Omega_m = \frac{8\pi G\rho_m}{6H^2}, \quad \Omega_\Lambda = \frac{8\pi G\rho_\Lambda}{6H^2} \quad (4)$$

so that Eq.(2) takes the form

$$\Omega_m + \Omega_\Lambda = 1. \quad (5)$$

The energy conservation for the KK universe becomes

$$\dot{\rho} + 4H(\rho + p) = 0.$$

Since we consider mixture of DE and DM (dust like), the above conservation equation does not hold globally. It should be decomposed into two non-conserving equations for DM and MHDE respectively, i.e.,

$$\dot{\rho}_m + 4H\rho_m = \Upsilon_4, \quad \dot{\rho}_\Lambda + 4H(\rho_\Lambda + p_\Lambda) = -\Upsilon_4, \quad (6)$$

where  $\Upsilon_4$  is the energy exchange term. The available three forms of energy exchange term in literature (Sun 2010) are given as

$$\Upsilon_1 = 3dH\rho_\Lambda, \quad \Upsilon_2 = 3dH(\rho_\Lambda + \rho_m), \quad \Upsilon_3 = 3dH\rho_m,$$

where  $d$  is a coupling constant which has a crucial role in the interpretation of interaction processes, i.e., either DM decays into DE or DE decays into DM depending on the sign of  $d$ .

The positive value of  $d$  stands for decay from DE to DM and negativity of  $d$  exhibits its reverse process. Moreover, the negative value of the coupling constant  $d$  would lead  $\rho_m$  to negative for far future when  $\Upsilon = \Upsilon_1$  and  $\Upsilon = \Upsilon_2$ . For  $\Upsilon = \Upsilon_3$  with negative  $d$ , it does not suffer such type of difficulties. But Pavon and Wang (2009) pointed out that GSLT favors the criteria of decays from DE to DM. Sun (2010) proposed a new form of energy exchange term

$$\Upsilon_4 = 3dH(\rho_\Lambda - \rho_m), \quad (7)$$

which is the best suitable model with the current observations. The value of  $d$  is assumed to be positive due to the reasons mentioned above. The

interesting feature of this interacting model is that it shows consistency with GSLT according to the arguments (Pavon and Wang 2009) both for early as well as late time.

In the following, we develop the modified version of holographic dark energy in KK cosmology. For this purpose, we consider a relationship of mass and radius of the Schwarzschild black hole in  $N + 1$  dimensions as (Myers 1987)

$$M = \frac{(N-1)A_{N-1}R_H^{N-2}}{16\pi G},$$

where  $A_{N-1}$  represents the area of unit  $N$ -sphere,  $R_H$  stands for the horizon scale of black hole. Also,  $G$  ( $N + 1$  dimensional gravitational constant) is associated with  $M_{N+1}$  ( $N + 1$  dimensional Planck mass) and the usual Planck mass  $M_{pl}$  in 4-dimensional spacetime through

$$8\pi G = M_{N+1}^{-(N-1)} = \frac{V_{N-3}}{M_{pl}^2},$$

where  $V_{N-3}$  is the volume of the extra-dimensional space. Thus the mass becomes

$$M = \frac{(N-1)A_{N-1}R_H^{N-2}M_{pl}^2}{2V_{N-3}}.$$

The mass related to the MHDE is given as (Myers 1987)

$$\rho_\Lambda = \frac{m^2(N-1)A_{N-1}L^{N-5}M_{pl}^2}{2V_{N-3}}.$$

For KK cosmology, we substitute  $N = 4$  and the value of  $A_3$  in the above expression, it follows that

$$\rho_\Lambda = \frac{3m^2\pi^2L^2}{8\pi G}. \quad (8)$$

The future event horizon  $L$  is defined as (Li 2004)

$$L = a(t) \int_a^\infty \frac{d\acute{a}}{H\acute{a}^2} = a(t) \int_a^\infty \frac{d\acute{t}}{a(\acute{t})}.$$

Its derivative with respect to time yields

$$\dot{L} = HL - 1. \quad (9)$$

For the de Sitter spacetime, the future event horizon becomes  $H^{-1}$  and  $\dot{L} = 0$ . Thus the future event horizon and apparent horizon coincide with each other for the spatially flat de Sitter universe and there is only one cosmological horizon.

Using Eqs.(4) and (9), we obtain time derivative of energy density

$$\dot{\rho}_\Lambda = -\rho_\Lambda H \left( -2 + \frac{2m\pi}{H^2 \sqrt{2\Omega_\Lambda}} + \Delta_G \right), \quad (10)$$

where  $\Delta_G \equiv \frac{G'}{G}$ ,  $\dot{G} = G'H$ , here prime denotes derivative with respect to  $\ln a(t)$ . Using EoS of DE,  $p_\Lambda = \omega_\Lambda \rho_\Lambda$ , in Eq.(6), we have

$$\dot{\rho}_m + 4H(1 + \omega_m^{eff})\rho_m = 0, \quad (11)$$

$$\dot{\rho}_\Lambda + 4H(1 + \omega_\Lambda^{eff})\rho_\Lambda = 0, \quad (12)$$

where

$$\omega_m^{eff} = -\frac{\Upsilon_4}{4H\rho_m}, \quad \omega_\Lambda^{eff} = \omega_\Lambda + \frac{\Upsilon_4}{4H\rho_\Lambda} \quad (13)$$

are effective equations of state for DM and MHDE respectively. Inserting the values of  $\dot{\rho}_\Lambda$  and  $\Upsilon_4$  in Eq.(12) and using Eq.(4), we obtain

$$\omega_\Lambda = -\frac{3}{2} + \frac{m\pi}{2H^2 \sqrt{2\Omega_\Lambda}} + \frac{1}{4}\Delta_G - \frac{3d}{4}\left(2 - \frac{1}{\Omega_\Lambda}\right). \quad (14)$$

This is the EoS for the MHDE which depends upon Hubble parameter  $H$ , dimensionless DE density  $\Omega_\Lambda$ ,  $\Delta_G$  and also two positive constant parameters  $m$ ,  $d$ .

According to the present status of the universe, it was observed that  $H_0 = 1$  (here 0 denotes the present value) (Paul et al. 2009),  $\Omega_\Lambda = 0.73$  and the value of  $\Delta_G$  lies in the range  $0 < \Delta_G \leq 0.07$  (Setare 2006a; Jamil et al. 2009). Also, Sun (2010) investigated that the value of interacting parameter  $d = 0.001$  is consistent with the analysis of (Pavon and Wang 2009) which shows consistency with the known properties of DE. However, the value of constant parameter  $m$  is still ambiguous. The best value of  $m$  obtained from observational type Ia supernovae is 0.21 (Huang and Gong 2004) while it leads to 0.61 from the X-ray gas mass fraction of galaxy clusters (Chang et al. 2006). Both of these values of  $m$  are taken within  $1 - \sigma$  error range. Also, it was consensus on the value of  $m = 0.91$  taken from combining the observational data of type Ia supernovae, cosmic microwave background

radiation and large scale structure within  $1 - \sigma$  error range (Zhang and Wu 2005). Later, its value  $m = 0.73$  was observed through combining the data of type Ia supernovae, X-ray gas and Baryon Acoustic Oscillation (Wu et al. 2008; Ma and Gong 2009).

Using these values of the parameters, especially of  $\Delta_G$  and  $m$ , we evaluate the value of EoS parameter which remains less than  $-1/3$ , i.e.,  $\omega_\Lambda \leq -1/3$  for  $m = 0.61, 0.73, 0.91$  (which follows the quintessence DE region). For  $m = 0.21$ , we have phantom DE era, i.e.,  $\omega_\Lambda < -1$ . These ranges of  $\omega_\Lambda$  describe the accelerating expansion of the universe through DE as a driving force. Thus the MHDE shows the transition from quintessence DE era to phantom DE era under certain assumptions.

The fractional energy density of MHDE is

$$\Omega_\Lambda = \frac{8\pi G\rho_\Lambda}{6H^2}. \quad (15)$$

Also, we have

$$\frac{2\dot{H}}{H^2} = -4(1 + \omega_\Lambda\Omega_\Lambda) + \Delta_G. \quad (16)$$

Taking derivative of Eq.(15) with respect to cosmic time and using Eq.(16), it follows that

$$\dot{\Omega}_\Lambda = \Omega_\Lambda H(6 + \omega_\Lambda\Omega_\Lambda - \Delta_G - \frac{m\pi}{2H^2\sqrt{2\Omega_\Lambda}}) \quad (17)$$

which can also be written as

$$\Omega'_\Lambda = \Omega_\Lambda(6 + \omega_\Lambda\Omega_\Lambda - \Delta_G - \frac{m\pi}{2H^2\sqrt{2\Omega_\Lambda}}). \quad (18)$$

This is the evolution equation for the MHDE which is an increasing function and represents the gradual increment of DE in the universe.

### 3 Generalized Second Law of Thermodynamics

According to this law, the sum of entropy of matter inside horizon and entropy of the event horizon cannot decrease with time (Izquierdo and Pavón 2006). The thermodynamics of cosmological scenario requires to consider



the universe as a thermodynamical system and it is related with the thermodynamics of black hole. The analysis of Hawking (1975) and Gibbons and Hawking (1977) about black hole implies that temperature of black hole horizon is inversely proportional to its mass. Bekenstein (1973) argued that the entropy of black hole would decrease with time and violate the second law of thermodynamics due to evaporation (in fact entropy of black hole horizon and horizon area are directly proportional). Due to this phenomenon, the entropy of the background universe increases with time. He concluded that the sum of black hole entropy and the background entropy must be an increasing quantity with respect to time.

In the following, we discuss the validity of the GSLT of interacting MHDE with DM with varying  $G$  in the KK universe enclosed by future event horizon. Gibb's equation (Izquierdo and Pavón 2006) relates the entropy of the universe including DE and DM inside the horizon with its pressure and internal energy (which is also known as first law of thermodynamics), i.e.,

$$TdS = pdV + dE,$$

where  $T$ ,  $S$ ,  $E$  and  $p$  are temperature, entropy, internal energy and pressure of the system respectively and  $V = \frac{\pi^2 L^4}{2}$  is the volume of the system. Splitting the above expression into MHDE and DM as

$$TdS_\Lambda = p_\Lambda dV + dE_\Lambda, \quad TdS_m = p_m dV + dE_m. \quad (19)$$

Since the number of particles inside the horizon is not conserved, so we assume chemical potential to be zero (Izquierdo and Pavón 2006). The temperature and entropy of horizon in KK universe become (Cai and Kim 2005)

$$T = \frac{1}{2\pi L}, \quad S_H = \frac{\pi^2 L^3}{2G}.$$

The rate of change of entropy of horizon is

$$\dot{S}_H = \frac{3\pi^2 L^2 \dot{L}}{2G} - \frac{\pi^2 L^3 \dot{G}}{2G^2}. \quad (20)$$

Thermodynamical and cosmological quantities are related as

$$p_\Lambda = \omega_\Lambda^{eff} \rho_\Lambda, \quad p_m = \omega_m^{eff} \rho_m, \quad E_\Lambda = \frac{\pi^2 L^4 \rho_\Lambda}{2}, \quad E_m = \frac{\pi^2 L^4 \rho_m}{2}. \quad (21)$$

Taking the derivative of Eq.(19) with respect to time, we have

$$\dot{S}_\Lambda = \frac{p_\Lambda \dot{V} + \dot{E}_\Lambda}{T}, \quad \dot{S}_m = \frac{p_m \dot{V} + \dot{E}_m}{T}. \quad (22)$$

Equations (20)-(22) lead to

$$\begin{aligned} \dot{S}_{total} &= 4\pi^3 L^4 [(1 + \omega_\Lambda^{eff})\rho_\Lambda + (1 + \omega_m^{eff})\rho_m](\dot{L} - LH) + \frac{3\pi^2 L^2 \dot{L}}{2G} \\ &\quad - \frac{\pi^2 L^3 \dot{G}}{2G^2}, \end{aligned} \quad (23)$$

where  $S_{total}$  is the sum of three entropies. Also, we know that

$$(1 + \omega_\Lambda^{eff})\rho_\Lambda + (1 + \omega_m^{eff})\rho_m = (1 + \omega_\Lambda)\rho_\Lambda + \rho_m \quad (24)$$

and

$$\frac{\rho_m}{\rho_\Lambda} = -1 + \frac{1}{\Omega_\Lambda}. \quad (25)$$

Finally, we obtain the following expression by using Eqs.(9), (24) and (25) in (23)

$$\dot{S}_{total} = \frac{3m^2 \pi^4 L^6}{8G} \left( -\omega_\Lambda - \frac{1}{\Omega_\Lambda} - \frac{m^2 \pi^2}{4H^4 \Omega_\Lambda^2} + \frac{4m\pi}{(2\Omega_\Lambda)^{\frac{3}{2}} H^2} (1 - \Delta_G) \right). \quad (26)$$

If we set the values of  $\Omega_\Lambda$ ,  $\Delta_G$  and  $\omega_\Lambda$  corresponding to all physical acceptable values of  $m$  discussed in the previous section, we find  $\dot{S}_{total} \geq 0$ . Thus the GSLT holds for KK universe containing interaction of MHDE and DM with varying  $G$  enclosed by the cosmological future event horizon.

## 4 Summary

We have explored interaction of MHDE with DM and generalized second law of thermodynamics in KK universe. Setare (2006b) obtained an expression of EoS for holographic dark energy by considering bulk brane Interaction and found that it may cross phantom divide  $\omega = -1$ . The same author (2009) pointed out that holographic Chaplygin gas model, with IR cutoff as future event horizon, behaves alike phantom fluid and cross the phantom divide in a Dvali-Gabadaze Porrati (DGP) braneworld framework. Liu et al. (2010) and

Bandyopadhyay (2011) found that EoS of the MHDE can cross the phantom divide from quintessence region to phantom region during the evolution by choosing future event horizon as the IR cutoff in a DGP braneworld scenario. It was showed (Setare and Shafei 2006; Setare and Vagenas 2008) that GSLT is respected under the special range of physical parameters for HDE with horizon's radius  $L$  measured from the sphere of horizon. Also, it was found (Dutta and Chakraborty 2010, 2011) for DGP braneworld interacting HDE and CDM, GSLT holds for the universe bounded by apparent horizon or future event horizon.

In this paper, we have taken interacting MHDE and DM with varying  $G$  in the platform of KK cosmology. Further, we assume that IR cutoff  $L$  in the MHDE as a future event horizon. We have found the EoS parameter  $\omega_\Lambda$  of MHDE depending on different parameters by using the interacting model suggested by Sun (2010). In particular, the constant parameter  $m$  plays the crucial role in evaluating  $\omega_\Lambda$ . It is found that  $\omega_\Lambda$  remains in the quintessence DE dominated era for  $m = 0.61, 0.73, 0.91$  and it enters into the phantom DE dominated era for  $m = 0.21$ . This means that the universe shows transition from quintessence DE to phantom DE era in the scenario of MHDE. With the expansion of the universe, it is determined that the varying  $G$  with time is an increasing function of time. Although the variations are negligibly small, so it does not affect on our major results.

We have also investigated the validity of the generalized second law of thermodynamics in this scenario. It turns out that GSLT holds for the specific choice of physical parameters. It is interesting to mention here that our results about evolution of the universe are consistent with the current observations (Liu et al. (2010), Setare (2009), Bandyopadhyay (2011) and also GSLT holds (Dutta and Chakraborty 2010, 2011) when EoS of MHDE can cross the phantom divide.

## References

- Armendariz-Picon, C., Damour, T., Mukhanov, V.: Phys. Lett. **B458**, 209(1999)
- Armendariz-Picon, C., Mukhanov, V., Steinhardt, P.J.: Phys. Rev. **D63**, 103510(2001)
- Bagla, J.S., Jassal, H.K., Padmanabhan, T.: Phys. Rev. **D67**, 063504(2003)

Bandyopadhyay, T.: Int. J. Theor. Phys. **50**, 3(2011)  
 Bekenstein, J.D.: Phys. Rev. **D7**, 2333(1973)  
 Bellini, M.: Nucl. Phys. **B660**, 389(2003)  
 Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Rev. **D 66**, 043507(2002)  
 Bergmann, P. G.: Int. J. Theor. Phys. **1**, 25(1968)  
 Biesiada, M., Malec, B.: Mon. Not. R. Astron. Soc. **350**, 644(2004)  
 Brans, C.H., Dicke, R.H.: Phys. Rev. **124**, 925(1961)  
 Cai, R.G.: Phys. Lett. **B657**, 228(2007)  
 Cai, R.G., Kim, S.P.: JHEP **0502**, 050(2005)  
 Caldwell, R.R.: Phys. Lett. **B545**, 23(2002)  
 Caldwell, R.R., Doran, M.: Phys. Rev. **D69**, 103517(2004)  
 Carroll, S.M., Hoffman, M., Trodden, M.: Phys. Rev. **D68**, 023509(2003)  
 Chang, Z., Wu, F.Q., Zhang, X.: Phys. Lett. **B633**, 14(2006)  
 Chiba, T., Okabe, T., Yamaguchi, M.: Phys. Rev. **D62**, 023511(2000)  
 Clifford, J. *D-branes* (Cambridge University Press, 2003)  
 Cohen, A., Kaplan, D., Nelson, A.: Phys. Rev. Lett. **82**, 4971(1999)  
 Davis, T. M. et al.: Astrophys. J. **666**, 716(2007)  
 Deffayet, C., Dvali, G.R., Gabadadze, G.: Phys. Rev. **D65**, 044023(2002)  
 Dirac, P.A.M.: Proc. Roy. Soc. Lond. A **165**, 199(1938)  
 Dutta, J., Chakraborty, S.: Gen. Relativ. Gravit. **42**, 1863(2010)  
 Dutta, J., Chakraborty, S.: Int. J. Theor. Phys. **50**, 2383(2011)  
 Fedeli, C., Moscardini, L., Bartelmann, M.: Astron. Astrophys. **500**, 667(2009)  
 Feng, B., Wang X.L. and Zhang, X.M.: Phys. Lett. **B607**, 35(2005)  
 Gibbons, G.W., Hawking, S.W.: Phys. Rev. **D15**, 2738(1977)  
 Gong, Y., Li, T.: Phys. Lett. **B683**, 241(2010)  
 Hawking, S.W.: Commun. Math. Phys. **43**, 199(1975)  
 Hsu, S.D.H.: Phys. Lett. **B594**, 13(2004)  
 Huang, Q.G., Gong, Y.G.: JCAP **08**, 006(2004)  
 Izquierdo, G., Pavón, D.: Phys. Lett. **B633**, 420(2006)  
 Jamil, M., Saridakis, E.N., Setare, M. R.: Phys. Lett. **B679**, 172(2009)  
 Kaluza, T.: *Zum Unitatsproblem der Physik* Sitz. Press. Akad. Wiss. Phys. Math. kl, 966(1921)  
 Kamenshchik, A.Y., Moschella, U., Pasquier, V.: Phys. Lett. **B511**, 265(2001)  
 Klein, O.: Zeits. Phys. **37**, 895(1926)

Li, M.: Phys. Lett. **B603**, 1(2004)  
 Liu, D.J., Wang, H., Yang, B.: Phys. Lett. **B694**, 6(2010)  
 Ma, Y.Z., Gong, Y.: Eur. Phys. J. **C60**, 303(2009)  
 Myers, R.C.: Phys. Rev. **D35**, 455(1987)  
 Nesseris, S., Perivolaropoulos, L.: Phys. Rev. **D73**, 103511(2006)  
 Nordtvedt, K.: Astrophys. J. **161**, 1059(1970)  
 Ozel, C., Kayhan, H., Khadekar, G.S.: Ad. Studies. Theor. Phys. **4**, 117(2010)  
 Padmanabhan, T.: Phys. Rev. **D66**, 021301(2002)  
 Paul, B.C., Debnath, P. S., Ghose, S.: Phys. Rev. **D79**, 083534(2009)  
 Pavón, D., Wang, B.: Gen. Relativ. Gravit. **41**, 1(2009)  
 Perlmutter, S. et al.: Astrophys. J. **517**, 565(1999)  
 Polchinski, J. *String Theory* (Cambridge University Press, 1998)  
 Ratra, B., Peebles, P.J.E.: Phys. Rev. **D37**, 3406(1988)  
 Riess, A.G. et al.: Astron. J. **116**, 1009(1998)  
 Sahni, V., Shtanov, Y.: JCAP **11**, 014(2003)  
 Setare, M. R.: Phys. Lett. **B642**, 1(2006a)  
 Setare, M.R.: Phys. Lett. **B642**, 421(2006b)  
 Setare, M.R.: Phys. Lett. **B641**, 130(2006c)  
 Setare, M.R.: Int. J. Mod. Phys. **D17**, 2219(2008)  
 Setare, M.R.: Int. J. Mod. Phys. **D18**, 419(2009)  
 Setare, M.R., Jamil, M.: Phys. Lett. **B690**, 1(2010a)  
 Setare, M.R., Jamil, M.: Europhys. Lett. **92**, 49003(2010b)  
 Setare, M.R., Jamil, M.: J. Cosmol. Astropart. Phys. **02**, 010(2010c)  
 Setare, M.R., Shafei, S.: J. Cosmol. Astropart. Phys. **0609**, 011(2006)  
 Setare, M.R., Vagenas, E.C.: Phys. Lett. **B666**, 111(2008)  
 Sharif, M., Khanum, F.: Gen. Relativ. Gravit. **43**, 2885(2011)  
 Sun, C.Y.: arXiv:1009.1214  
 Susskind, L.: J. Math. Phys. **36**, 6377(1995)  
 Umez, K., Ichiki, K., Yahiro, M.: Phys. Rev. **D72**, 044010(2005)  
 Wagoner, R. V.: Phys. Rev. **D1**, 3209(1970)  
 Weinberg, S.: Rev. Mod. Phys. **61**, 1(1989)  
 Weinberg, S.: *The Cosmological Constant Problems*, arXiv:0005265  
 Wess, J., Bagger, J. *Supersymmetry and Supergravity* (Princeton University Press, 1992)  
 Wesson, P.S.: Gen. Relativ. Gravit. **16**, 193(1984)  
 Wesson, P.S.: *Spacetime-Matter Theory* (World Scientific, 1999)  
 Wu, Q., Gong, Y., Wang, A., Alcaniz, J.S.: Phys. Lett. **B659**, 34(2008)

Zhang, X., Wu, F.Q.: Phys. Rev. **D72**, 043524(2005)  
Zhang, X., Wu, F.Q., Zhang, J.: J. Cosmol. Astropart. Phys. **01**,  
003(2006)